Using Optimization Procedures to Calculate Parameters of MEMS Macromodels

Anatolii Petrenko, Volodymyr Ladogubets, Oleksandr Beznosyk, Oleksii Finogenov

Abstract – **The approaches to get a circuit equivalent of microelectromechanical systems' (MEMS) mechanical components are considered. The usage of the circuit design software optimization procedures to adjust macromodels' parameters obtained as a result of the source equivalent circuit reduction is suggested. An example is considered and the simulation results are given for the problem of finding eigenfrequencies of a beam. The recommendations to choose optimization methods are given.**

Keywords – **ALLTED, ANSYS, MEMS, optimization, simulation.**

I. INTRODUCTION

Simulation of the dynamic processes which take place in modern electronic devices requires a high accuracy of the models used. This circumstance is especially important when simulating heterogeneous by their physical nature devices that is specific for modern microelectromechanical systems (MEMS). However, the most widespread CAD systems use, generally, one-aspect simulation, for instance, Spice for electronic circuit analysis, or Adams for virtual simulation of the complex machines and mechanisms that makes it difficult both getting models of the same accuracy order and joint simulation of the all object's parts. It should be noted that in the most cases simulation of the object in whole is impossible since heterogeneity of the mathematical model's bases for the subsystems of different nature that complicates integration of the different CAD software simulation results.

To avoid these problems, it is reasonable to use single software to simulate an object but this requires presentation of the subsystems of different physical nature in the basis of a CAD system chosen. For the mechanical systems, there is a method of electromechanical analogies allowing getting a mathematical model of a mechanical object as an equivalent electrical circuit from a finite-element description. In this case, the accuracy of an equivalent circuit obtained matches the accuracy of a finite-element description [1].

The requirements of a high simulation accuracy leads to a rather small mesh for the finite-element method that results in the increase of the considered equivalent circuit size and make difficulties for its using as a model. To eliminate this disadvantage, it is possible to use methods of mathematical model dimension reduction [2]. However, the mathematical reduction methods dealing directly with the equation systems result in a description that could not be presented as an equivalent circuit. Therefore, it seems more promising to use reduction methods based on the Y-Δ transformation, or methods of building macromodel as an n-terminal [3]. Such reduced equivalent circuits have a number of advantages including compactness of the description and the possibility to

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formalize the algorithm to obtain them.

II. Y-Δ TRANSFORMATION BASED CIRCUIT DIMENSION REDUCTION METHODS

The essence of the methods based on the Y-Δ transformation consists in the following. Let's *i*-th node and its *k* neighbours are located as shown on Fig. 1. Then, the component equation of *i*-th row looks like:

$$Y_i V_i - y_1 V_1 - y_2 V_2 - \ldots - y_n V_n = 0, \qquad (1)$$

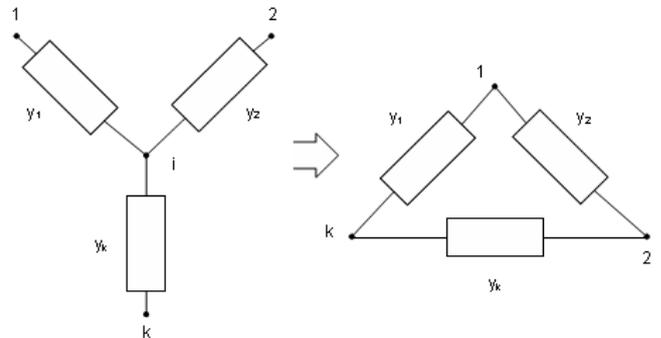where $Y_i = \sum_{j=1}^{k} y_j$.

Fig. 1 A work node of RLC circuit. Addition of a conductance between node 1 and nodes next to *i*-th.

To exclude V_i from (1), that is equivalent to excluding *i*-th node, let's define V_i as:

$$V_i = \left(\sum_{j=1}^{k} y_j V_j \right) \Big/ Y_i \qquad (2)$$

and replace V_i with (2) at *k* equations, where it is present. Let's consider a first node, which will be next to *i*-th, and its equation looks like:

$$\left(\bar{Y}_1 + y_1 - y_1^2 / Y_i \right) V_1 -$$
$$- \left(\sum_{j=2}^{k} y_1 y_j V_j \right) \Big/ Y_i - \sum_{\substack{r=1 \\ r \neq i}}^{k1} y_r V_r = 0 \qquad (3)$$

where $\bar{Y}_1 = \sum_{\substack{r=1 \\ r \neq i}}^{k1} y_r$ is a sum of all node one's conductances excluding *i*-th node, *k1* is a number of nodes connected to node 1.

Eq. (3) can be simplified:

$$\left(\bar{Y}_1 + \left(\sum_{j=2}^{k} y_1 y_j\right)\Big/ Y_i\right)V_1 -$$

$$-\left(\sum_{j=2}^{k} y_1 y_j V_j\right)\Big/ Y_i - \sum_{\substack{r=1 \\ r \neq i}}^{k1} y_r V_r = 0$$

Note that it is equivalent to adding *k-1* new elements between first node and *k-1* former neighbours of *i*-th node on Fig. 1.

For any two nodes next to *i*-th, for example *a* and *b*, excluding of *i*-th node leads to adding a new element between these nodes. Conductance of this element will be equal to:

$$y_{ab} = (y_a y_b)/Y_i$$

So, repeating this process for all *k* neighbours of *i*-th node, we remove it thereby.

For each node in the circuit, two time constants could be defined: $\tau_{RCi} = C_i / G_i$ and $\tau_{LCi} = \sqrt{C_i / B_i}$, where $C_i = \sum_{j=1}^{k} c_j$ is a sum of all capacitances connected to node *i*, $B_i = \sum_{j=1}^{k} b_j$ is a sum of all reactive conductances (the value reverse to inductance) connected to node *i*, and $G_i = \sum_{j=1}^{k} g_j$ is a sum of all conductances connected to node *i*. So, *i* node's time constant is:

$$\tau_i = \max(\tau_{RCi}, \tau_{LCi}).$$

i-th node is considered to be fast if:

$$\tau_i < \tau_{\min} = 2\pi / \omega_{\max},$$

where $\tau_{\min}$ is a user-defined time constant, which depends on the maximal frequency of the circuit – $\omega_{\max}$. Note that the $\tau_{\min}$ constant is proportional to $1/\omega_{\max}$. Thus, if *i*-th node is a fast one it can be removed from the circuit as described above. More details about reduction algorithms based on the Y-Δ transformation are in [4,5,6].

In practice, the lower eigenfrequencies, which are work ones for mechanical systems, is of interest usually. Therefore, a compromise between accuracy and dimension of the received models can be reached by the variation of $\tau_{\min}$.

III. METHOD OF THE CONSTRUCTION OF A MACROMODEL AS AN N-TERMINAL

The main idea of the method is the construction of a circuit macromodel as a four-terminal circuit (n-terminal in a general case) as:

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} Y_{aa} & Y_{ab} \\ Y_{ba} & Y_{bb} \end{bmatrix}\begin{bmatrix} U_a \\ U_b \end{bmatrix}, \quad (4)$$

where I_a, U_a are current and voltage at the macromodel's input, I_b, U_b are current and voltage at the macromodel's output.

It is suggested to obtain Eq. (4) directly by the common circuit matrix (for example, conductance matrix *Y*) according to:

$$Y_{tp}(i) = \frac{1}{\Delta_{aa,bb}}\begin{bmatrix} \Delta_{bb} & -\Delta_{ba} \\ \Delta_{ab} & \Delta_{aa} \end{bmatrix},$$

where Δ_{ij} are algebraic adjuncts of the i, j element of the source matrix.

It is possible not to calculate separately the required algebraic adjuncts but to get all of them by means of the source matrix inversion procedure, with elements defined at a fixed frequency ω_0. So, if the numeric values of elements of the inverse matrix Y^{-1} has been found:

$$Y^{-1} = \begin{bmatrix} g_{11} & g_{21} & \cdots & g_{n1} \\ g_{12} & g_{22} & \cdots & g_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ g_{1n} & g_{2n} & \cdots & g_{nn} \end{bmatrix}, \quad (5)$$

then it is possible to find easy a circuit macromodel as a four-terminal (4). For that, in the case of a four-terminal with ports *a* and *b*, it is needed to select four elements of the inverse matrix (5) $g_{aa}, g_{bb}, g_{ab}, g_{ba}$ and to calculate values:

$$g_{aa,bb} = g_{aa}g_{bb} - g_{ab}g_{ba}.$$

Parameters of the reduced model could be found from:

$$Y_{aa} = g_{bb} / g_{aa,bb};$$
$$Y_{ab} = -Y_{ba} = g_{ba} / g_{aa,bb};$$
$$Y_{bb} = g_{aa} / g_{aa,bb}.$$

Since the parameters of a macromodel are determined by a sum of the real and imaginary parts:

$$Y_{ij} = a_0 + ia_1,$$

circuit macromodel equations could be presented as:

$$Y_{tp} = \begin{bmatrix} Y_{aa0} & Y_{ab0} \\ Y_{ba0} & Y_{bb0} \end{bmatrix} + i\begin{bmatrix} Y_{aai} & Y_{abi} \\ Y_{bai} & Y_{bbi} \end{bmatrix}. \quad (6)$$

If $a_o > 0$, $a_1 > 0$, the respective component of the circuit macromodel could be presented as a parallel connection of conductance and capacitance:

$$G_{i,j} = \mathrm{Re}(Y_{i,j}) \text{ and } C_{i,j} = \mathrm{Im}(Y_{i,j})/\omega_0.$$

If $a_o > 0$, $a_1 < 0$, the respective component of the circuit macromodel could be presented as a parallel connection of conductance and inductance:

$$G_{i,j} = \mathrm{Re}(Y_{i,j}) \text{ and } L_{i,j} = 1/(\mathrm{Im}(Y_{i,j})/\omega_0).$$

A real part for the stable passive macromodels has to be positive ($a_o > 0$) while an imaginary part could be negative. In this connection, circuit interpretation of matrix

elements could be presented taking into account their imaginary part's sign.

Such the simplest circuit interpretation of the model (6) equations leads to the appearing of resistances at the macromodel connected between input *a* and output *b* nodes and a common node of the four-terminal, and, respectively, voltage dividers, which distort an amplitude of the output signal. So, there is a sense to select a priori a circuit model, for example, shown on Fig. 2, and to find ratios between parameters of its components and parameters of the mathematic macromodel (6). Similarly, such ratios could be deduced also for other selected circuit interpretations of macromodels obtained [3].
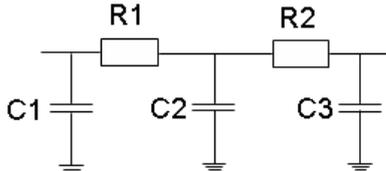

Fig. 2 Circuit macromodel chosen.

The important feature of the descriptions obtained as equivalent circuits is a possibility to use optimization procedures for accurate adjustment of the macromodel components' values. However, the method based on the obtaining macromodel as an n-terminal has good approximation characteristics of the source circuit at the separate frequencies only, and the repetitive macromodel construction is required due to their changing. Optimization of the source macromodel is complicated by the fact that there are no parameter variation limits. So, the optimization methods for macromodels obtained by means of the methods based on Y-Δ transformation will be considered later in this article.

IV. EXAMPLE DESCRIPTION

Let's consider the possibilities of using CAD software optimization methods for mechanical component models and further usage of those models by the example of a problem of finding eigenfrequencies of a beam [7] (Fig. 3).
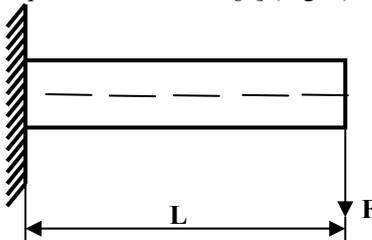

Fig. 3 Beam scheme.

The construction in question represents a beam working on a bending. The left end of the beam is fixed motionlessly, right one is free. Force *F* is applied to the right end perpendicularly to the beam axe.

To construct a mathematic model, ANSYS Multiphysics v.10.0 software developed by ANSYS Inc. [8] was used. ANSYS simulation took place under the following parameters: L = 25 mkm; beam cross-section is a square one with height of 3 mkm and width of 2 mkm. Beam material properties: coefficient of elasticity E=$2 \cdot 10^{11}$ Pa = 0.2 N/mkm^2; Poisson coefficient μ = 0.3; material density ρ = $6 \cdot 10^3$ kg/m^3 = $6 \cdot 10^{-9}$ mg/mkm^3.

For discretization of the beam, ANSYS library's BEAM3 finite elements have been used. Every beam element has two nodes (left/right) with three degrees of freedom in each one: coordinates *x*, *y* in a plane of motion and a corresponding beam cross-section's turn angle *θ*. All the elements have identical length of 0.5 microns. So, the design model consists of 50 beam elements and 51 nodes.

Using the method of electromechanical analogies, it is possible to get an equivalent electronic circuit [9]. For the considered example, the equivalent circuit is presented on Fig. 4.
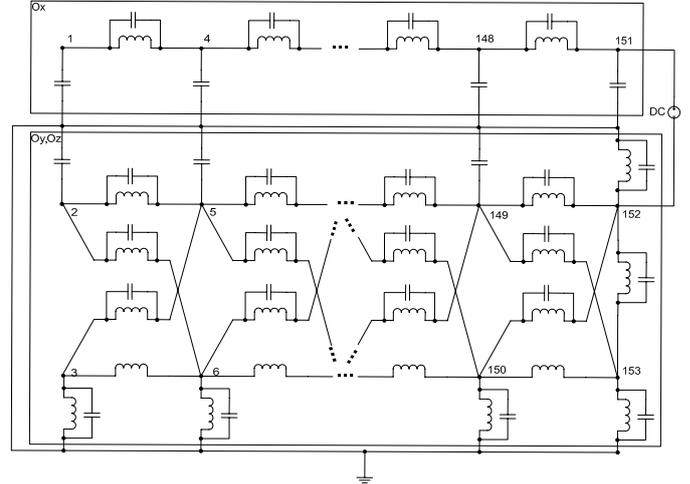

Fig. 4 Equivalent circuit.

The four lower eigenfrequencies of the construction in the range of 14 MHz up to 120 MHz are defined by computation. The results of ANSYS Multiphysics frequency analysis as well as the results of simulation of the equivalent and reduced under the different values of τ_{min} circuits in NetALLTED [10] are given in Table 1.

TABLE 1

FREQUENCY ANALYSIS RESULTS

	ANSYS results	ALLTED results			
		Source circuit	Reduced circuit		Optim. circuit
τ_{min}	-	-	$2*10^{-4}$	$4*10^{-4}$	$8*10^{-4}$	$8*10^{-4}$
Node number	-	154	27	15	11	11
Element number	-	862	256	94	66	66
Reduction by nodes, %	-	-	82.468	90.260	92.857	92.857
Reduction by elements, %	-	-	70.303	89.095	92.343	92.343
1st peak, MHz	18.745	18.745	18.788	18.724	18.955	18.728
2nd peak, MHz	50.890	50.890	51.028	50.458	52.613	50.909
3rd peak, MHz	97.860	97.859	97.955	95.937	101.75	97.873
4th peak, MHz	115.49	115.49	114.93	112.88	107.2	115.42
Maximal error, %	-	-	0.48	2.26	7.18	0.09*

* - total accumulated error by all four frequencies is 0.2%.

V. Optimization

It is obvious that the accuracy of a model obtained under $\tau_{min}=8*10^{-4}$ is rather low but there are few nodes and elements in the reduced circuit. For more accurate simulation it is possible either to make use of a reduced circuit obtained under the lesser values of τ_{min} but in this case the size of an equivalent reduced circuit increases, or to adjust a model in the range in question by help of the optimization methods. The advantages of the approach based on optimization of the circuit parameters are a small size of the equivalent reduced circuit as well as, in a narrow range of work frequencies, the possibility to get required frequencies with a high accuracy. The disadvantage is a necessity to make the additional analysis of the circuit in order to find the most sensitive elements. At the rather small size of a reduced circuit the sensitivity of each element increases making difficulties both for defining variable parameters' changing ranges and for optimization methods to find global optimum.

For example, for circuit obtained under $\tau_{min}=8*10^{-4}$, using sensitivity analysis presented in the ALLTED software, eight elements have been separated, whose changing influences in the most significant way on the frequencies of the range in question (14-120 MHz). An objective function for the circuit given could be formed in the package input language as follows:
OF ERROR1=F1(18.745,50.89,97.859, 115.49/T1,T2,T3,T4), where T1-T4:
FIX T1=MAXA(db.K1,15,25);
FIX T2=MAXA(db.K1,40,60);
FIX T3=MAXA(db.K1,85,105);
FIX T4=MAXA(db.K2,106,120);
MAXA directive allows defining time or frequency when the argument (db.K1 in our case) reaches its maximum value in the specified time or frequency range. As an error function the following one:

$$F1 = \sum_{i=1}^{n} (PV_i - x_i)^2$$

has been chosen where x_i is an output characteristic defined by FIX, INT or FUNC directives [10], any symbolic parameter or linear component; PV_i is a value of x_i characteristic (argument) that should be reached as a result of optimization.

Due to a large number of local extremums, the random search method with search interval reducing has been used to optimize parameters. A sample was 500 estimations of the objective function per iteration. The results of simulation of the circuit with the parameters after optimization are given in Table 1. The number of the objective function estimations performed is 2562. To reduce simulation time at the large size of a sample, it is recommended to use possibilities of the multiprocess realization of the algorithm [11].

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VI. Conclusion

The method to form circuit realizations of mechanical (non-electrical) object models presented in the paper allows simulating such devices by means of the circuit design software intended for using in the field of electronics. In the same time, equivalent circuit simulation accuracy does not yield to ANSYS simulation accuracy. To reduce dimension of the equivalent circuits obtained, it is suggested using Y-Δ transformation based RLC-circuit reduction methods. The accuracy of the reduced equivalent circuit could be reached by both reduction parameter variation and using element parameter adjustment by means of the circuit design software optimization procedures. When dealing with small-sized reduced equivalent circuits, it is necessary to take into account a large elements' sensitivity. In this case, an objective function has a large number of local extremums leading to difficulties during the optimum solution search process. So, for the optimization purposes, it is recommended to choose random search methods with a large sample size.