DESIGN METHODOLOGY AND WORKFLOW FOR MEMS DESIGN

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Abstract - **In this paper MEMS models for different abstraction levels (from device level till system level) are considered. The workflow for MEMS design as a whole is presented.**

Keywords - **FEM/FDM Model, ROM (Reduced Order Model), MEMS system-level model.**

I. INTRODUCTION

Microelectromechanical Systems (MEMS) are components with micron-scale moving parts made from the materials and processes of microelectronics fabrication. They are good example of on-chip integration of electronics, microstructures, microsensors and microactuators. Accurate simulation of MEMS requires accurate modeling of the effects of mechanical and damping forces, electrostatic forces and intrinsic stresses, heat transfer, thermal expansion, piezoelectric stresses and so on.

Modern methodology of MEMS design confirms that MEMS as a whole can be investigated only at the higher abstraction levels such as schematic and system levels for which still accurate macromodels can be used. But at component or device levels the physical behavior of three-dimensional continuums is described by partial differential equations (PDE) which are typically solved by Finite Element or Finite Difference Element Methods (FEM or FDM), being provided by ANSYS –like software. Component level simulations are classified in single domain and coupled domain simulations and they are very computer time- consuming.

The goal of this paper is to consider methods of automatically obtaining macromodels of MEMS and their mechanical or non-electric components from ANSYS models as equivalent electric circuits or low order differential ordinary equations for further use in circuit design software. This can be done by using different model order reduction techniques which were developed in recent years.

When developing the modern MEMS, the possibility to use a single environment to simulate objects where different physical processes, such as electrical, mechanical, optical, thermal etc. take place plays an important role. This requires representing different subsystems of the initial MEMS as equivalent models of the same physical nature that will allow joining them for solution in a single computational process. Then complete behavioral model of the whole MEMS and its subsystems can be compiled or in VHDL-AMS language (as sets of ODE) or SPICE-like language (as equivalent electric circuits).

Microsystems design exploits various analytical and numerical methods for virtual prototyping of MEMS. It also exploits libraries of models of electromechanical, optical and microfluid components, among which are springs, bulks, buffers, capacitors, inductances, operational amplifiers, transistors and so on. Three basic possible steps of the entire design procedure are illustrated below.

II. FEM/FDM MODEL

MEMS typically involve multiple energy domains, such as kinetic energy, elastic deformation, electrostatic or magnetostatic stored energy and fluidic interactions. Much of the difficulty in the modeling of MEMS devices is due to the tight coupling between the multiple energy domains. Individual physical effects are governed by partial differential equations (PDE), typically nonlinear. When these equations become coupled, the computational challenges of highly meshed numerical simulation become formidable. The use of FEM codes in this field remains limited for two reasons. First, the use of FEM codes to simulate MEMS devices is prohibitively cumbersome, expensive, and time consuming. Consequently, it is very expensive to close the loop on an FEM model of a device to allow for the design of feedback control laws or to use the model in system-level simulations. As a result, FEM models are mostly used to analyze the performance of MEMS components and to couple their multiphysics effects. Second, FEM models use numerous sets of variables to represent the device state. This approach makes the process of mapping the design space complex. Also, the relationship between each of these variables and the overall component performance is not clear to designers.

By reading binary files of the FEM software like ANSYS it is possible to assemble a MEMS component state-space model in the form of systems of first order or second order ordinary diffential equations (ODE)

$$E_r z' + A_r z = B_r f, \qquad Y=C_r z \tag{1}$$

$$Mx'' + Dx' + Kx = Bf, \qquad Y=Q^T x + R^T x', \tag{2}$$

where A_r, E_r ,C_r,B_r, M ,D, K, B, C- are the system matrices, B_r,B are the input and the C_r,C -output matrices, f is input force. In mechanics matrices M, D and K are known as the *mass, damping* and *stiffness* matrices correspondingly.

In (1) the state space vector z is defined through the unknowns deflections $u(x,t)$ and pressures $p(x,y,t)$ into the node points being automatically generated in MEMS structure:

$$z = [u_1 ... u_N \frac{\partial u_1}{\partial t} ... \frac{\partial u_N}{\partial t} p_{11} ... p_{MN}]^T \tag{3}$$

By defining

$$E_r = \begin{bmatrix} D & M \\ M & 0 \end{bmatrix} \quad A_r = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \quad B_r = \begin{vmatrix} B \\ 0 \end{vmatrix} \quad C_r = \begin{vmatrix} Q \\ R \end{vmatrix} \quad z = \begin{vmatrix} x' \\ x'' \end{vmatrix}$$

second equations (2) can be transferred to the first (1).

III. ROM (REDUCED ORDER MODEL)

It would be easier and more intuitive for the designer to explore the design space if the MEMS model had only a few variables with a clear relationship between them and the overall device performance. *Reduced-order models(ROM)*, also called *macromodels*, lend themselves very well to these purposes. MEMS models resulted in seek to capture the most significant characteristics of a device behaviour in a few variables governed by few ordinary-differential equations of motion.

The main idea behind the macromodel is that the number of ODE's needed to simulate the system has been reduced from perhaps many thousands in the case of the full FEM simulation, to just a few basis function coordinates. Thus the macromodel simulation can be very efficient computationally compared to the FEM model. A designer can use the FEM model for different component geometry and materials trying and the ROM model for investigation of different input forces effect.

III.1 KRYLOV/ARNOLDI ROM

One of the many popular methods for this kind of reduction is by using Krylov subspaces techniques. Since the order of the equation system (1) is quite large we need a lower order state-space model. Using Krylov/Arnoldi method, we reduce a state space equation

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bv(t) \\ Y(t) &= C^T X(t) \end{aligned} \tag{3}$$

to

$$\begin{aligned} \dot{x}(t) &= A_r x(t) + B_r v(t) \\ y(t) &= C_r^T x(t) \end{aligned} \tag{4}$$

The dimention of matrices A_{red}, B_{red} and C_{red} and the internal state vectors x_{red} are significantly smaller after model reduction for the same size of input vector f and output vector y. The node points being automatically generated in MEMS structure have to be partitioned into boundary nodes (related to the terminals at the model interface) and internal nodes (related to internal states of the reduced model).

Krylov's metod is based on approximation of system transfer function by moment matching. For linearized state-space model (3), perform a Laplace transform to obtain its frequency domain transfer function *F(s)* and expand it into Taylor series as

$$F(s) = -\sum_{k=0}^{\infty} m_k s^k \,, \tag{5}$$

where $m_k = -C^T A^{-k} (A^{-1} B)$.

The problem of constructing a reduced-order model of size q that approximates the input-output behavior of (3) can be stated as follows: determine the reduced linear state space system (4) such that the transfer function of the reduced system can approximate the transfer function of the original system.The transfer function of such a reduced system is

$$F_{red}(s) = -\sum_{k=0}^{\infty} m_{\kappa,k} s^k \,, \tag{6}$$

where $m_{r,k} = -C_{red}{}^T A_{red}{}^{-k} (A_{red}{}^{-1} B_{red})$.

We can approximate *F(s)* by $F_{red}(s)$ by matching the first q moments of *F(s)*, i.e. we will have $m_{r,k} = m_k$ for $k = 0, 1, \dots,$ q-1.

However, quite often practitioners use the Arnoldi process as it is more numerically robust and allows us to *preserve stability and passivity* of the original FEM model without extra computational cost. To apply Arnoldi method to do moment matching for the system characterized by $\{A, B, C\}$ matrices, first define $A_1 = A^{-1}$ and $b = A^{-1}B$, and then apply Arnoldi method on A_1, b and q. The results are H_q and L_q. In Arnoldi method $L_q = [l_1 \ . \ \dots, \ l_q]$ is the basis for orthogonal Krylov subspace and $H_q = (h_{i,j})$ is a replication of the original system's matrix A into the Krylov subspace taking into account Lq. The matrices possess the following properties: $L_q^T L_q = 1$ and $AL_q = L_q H_q + h_{q+1,q} I_{q+1} e_q^T$ .With these operations, the kth moment of the system (3) can rewritten as $m_k = -C^T A^{-k} (A^{-1} B) = C^T A_1^k b = \lVert b \rVert_2 C^T L_q H_q^k e_1 = \lVert b \rVert_2 C^T L_q H_q^{k+1} H_q^{-1} e_1$ From comparison of left and right parts it can be seen that if we choose $A_{red} = Hq^{-1}$, $B_{red} = \lVert b \rVert_2 H_q^{-1} e_1$ and $C_{red} = L_q^T C$, then $m_k = m_{r,k}$ $(k=0,1,\dots,q-1)$ can be satisfied.

Using Krylov/Arnoldi approach, only a postprocessor is necessary to generate a system-level model in one of the well-established model description languages: pure C code, HDL-A , MAST and the new standardized VHDL-AMS which are supported by powerful system simulators. Such approach was investigated by T. J. Su , R. R. Craig, E. B. Rudnyi, J. G. Korvink, J. Lienemann, A. Greiner, J. Chen, Sung-Mo Kang, Jun Zou,Chang Liu , José E , M. Schlegel, F. Bennini, J. Mehner, G. Herrmann, D. Müller, W. Dötzel Reitz, Sven, Christian D¨oring, Jens Bastian, Peter Schneider, Peter Schwarz, and Reinhard Neuland, etc. and it was implemented in INTEGRATOR system of CoventorWare, MEMS Pro , mor4ansys, MEMS MODELER of MEMSCAP, etc. The reduced model of order 20- 30 was able usually to describe the original model of dimension up to 100 000 with an accuracy of a few percent.

III.2 MODAL DECOMPOSITION ROM

This method uses the solution of PDE *L(u)=f* equations being approximated by series expansion of time varying coefficients $a_i\,(t)$ and spatially varying basis functions $b_j(x)$:

$$\tilde{u}(x) = \sum_{i=1}^{n} a_i(t)\, b_i(x) \tag{8}$$

As it is well known for the Galerkin's method the PDE residual (*L(u)-f*) had be orthogonal to each of the basic functions : $(a_i, L(\tilde{u}) - f) = \int a_i^T (L(\tilde{u}) - f)dx = 0, \quad i = 1, N$.

Recently, macromodels based on three proper orthogonal decomposition (POD) methods including Singular Value Decomposition (SVD) , Karhunen-Loève decomposition (KLD) and neural networks-based generalized Hebbian algorithm (GHA) have been developed. For example, it was proposed to determine $a_i(t)$ and $b_j(x)$ through the eigenvalues and the eigenvectors (modes) of the stiffness matrix.

The eigenvectors $b_j(x)$ and eigenfrequencies ω_i of the considered modes i can be taken from the modal analysis of the mechanical structure. All missing parameters of the ROM can be derived from a detailed fully coupled FEM model of the MEMS component in a highly automated manner. For MEMS is is usually sufficient a few eigenmodes to accurately describe dynamical response of the system.

The choice of orthogonal basis functions $b_j(x)$ can be done also in the following way. First the MEMS dynamics are simulated using a slow but accurate technique such as FEM or FDM. A set of runs may be used to suitably characterize the operating range of the device. The spatial distributions of each state variable u(x,t) are then sampled at a series of N_s different times during these simulations, and the sampled distributions are stored as a series of vectors, $\{u_i\}$, where each corresponds to a particular "snapshot" in time. Now suppose we would like to pick orthogonal basis N functions $\{b_1,b_2,\ldots,b_N\}$ in order to represent the observed state distributions as closely as possible. One way to do this is to attempt to minimize a least squares measure of the "error" distances between the observed states and the basis function representation of those states. It turns out that this can be accomplished quite simply by taking the SVD (Singular Value Decomposition) of the matrix U, whose columns are u_i. The SVD gives $U = V\Sigma W^T$, where $\Sigma = diag(\sigma_1,\ldots,\sigma_N)$ is a diagonal matrix, V and W are orthonormal matrices of eigenvectors, , and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_N \geq 0 \quad N < N_a$

The approach in hand was investigated by L. D. Gabbay, Jan Mehner, S. D. Senturia, H. Tilmans, M. D. Graham, I. G. Kevrekidis, Bennini, Lynn D., Jan E. Mehner, Stephen D, Fouad, Jan Mehner, Wolfram D¨otzel,Torsten M¨ahne_, Kersten Keh, Bertram Schmidt , etc. and it was implemented as the ROM-Tool available in ANSYS/Multiphysics since Release 7 (ROM144). It gives in a factor of 40 speed up in computation time over the finite-element model while suffering pull-in time errors of less than 2%.

III.3 Circuit ROM

Taking into account relations between displacements x, velocities v and accelerations a : $a = dv/dt, \quad x = \int vdt$, it is possible to present an equation (2) in the form

$$\frac{d}{dt}(Mv) + Dv + \int Kvdt = F(t) \text{ or } \tilde{C}\dot{v} + \tilde{G}v + \tilde{L}v = F(t), \quad (9)$$

where $\tilde{C} = M, \tilde{G} = D, \tilde{L} = K$ – are equivalent matrices of capacitances, conductance and inductances. The elements of matrices C, G, L are formed from the elements of the mass, damping and stiffness matrices in the following ways:

$$C_{ij} = -m_{ij}, \quad i,j = 1(1)N, \quad i \neq j$$

$$C_{ii} = \sum_{j=1}^{N} m_{ij}, \quad i = 1(1)N.$$

$$L_{ij} = -1/k_{ij}, \quad i,j = 1(1)N, \quad i \neq j$$

$$L_{ii} = 1/\sum_{j=1}^{N} k_{ij}, \quad i = 1(1)N.$$

$$G_{ij} = -d_{ij}, \quad i,j = 1(1)N, \quad i \neq j$$

$$G_{ii} = \sum_{j=1}^{N} d_{ij}, \quad i = 1(1)N,$$

where N is a number of equations or nodes of the MEMS structure. In this way a capacitance-inductance-conductance circuit model shall be constructed which reflects correctly mass, damping and stiffness matrices . For reduction of such model size the Y/Δ transformation is usually used by removing "high frequencies" inner nodes with the maximum value of time constant. In this case (as in the previous ones) it is necessary to find a compromise between the model reduction order and its exactness (adequacy).

The application of Y/Δ transformation does not require modification of existent silulation tool, as end results is a reduced circuit unlike the existing approaches (for example, in CoventorWare), where it is represented by the reduced system of differential equations. Thus, the design of electric and non-electric MEMS components can be done by one circuit simulation.

The circuit ROM approach was investigated by R. H. MacNeal, HsuJ.L., Vu-Quoc,.PelzG., Bielefeld, G.Zimmer, Petrenko A.I., Ladogubets V.V., Fenogenov A.D.,Besnosik A.U., etc. and it was implemented in the Ukrainian circuit simulation package NetALLTED (ALL TEchnologies Desinger) , which was developed not only for simulation and analysis, but for processing project procedures such as parametric optimization tasks; optimal tolerance assignments; centering availability regions; yield maximization; for design of Nonlinear Dynamic Systems composed of either/and electronic, hydraulic, pneumatic, mechanical, electromagnetic, and other elements (http://allted.kpi.ua/). This approach provides more than 99% reduction of elements and node numbers. Say for the accelerometer plate only 6 nodes are left from initial 1883 nodes.

IV. MEMS System-Level Model

As it was mentioned before a ROM any type can be used during the MEMS design for different input functions, resulting in enormous saving in the computational time. However, if an engineer would like to change the geometry or materials properties of MEMS in order to optimize its performance, a new ROM should be generated from modified FEM model after the changes are incorporated in it. So two MEMS models (FEM and ROM) had to be coupled to get an effective MEMS system-level model.

Say a capacitive RF Switch may be simulated at system level by nonlinear capacitance

$$C_{eq} = C_o + (C_{L/2} - C_0)(1 - e^{-\tau t}), \quad (10)$$

where C_0 is the smallest capacitance in the absent of voltage V, $C_{L/2}$ is the largest capacitance ,when a plate center displacement $w(\frac{L}{2},t)$ is calculated from the ROM macromodel equations:

$$C_{L/2} = \frac{\varepsilon_0 A_C}{d_e - w(\frac{L}{2},t)}. \quad (11)$$

In (11) A_C is the bottom electrode area; d_e is an equivalent gap ($d_e = d_0 + \frac{d_1}{\varepsilon_1} + \frac{d_{ins}}{\varepsilon_{ins}}$); ε_0 is the absolute dielectric permittivity of the vacuum, ε_1 is the relative dielectric permittivity of the poly silicon, ε_{ins} is the relative dielectric permittivity of the insulator; *w* is a plate diflaction. Parameter τ in (10) can be calculated through the plate center displacement $w(\frac{L}{2},t)$ and its velocity $v(\frac{L}{2},t) = w'(\frac{L}{2},t)$ in the following way $\tau = \frac{1}{3} w(\frac{L}{2},t) / v(\frac{L}{2},t)$.

The electrostatic force acting on the capacitor surfaces is the Coulomb force:

$$\tilde{F}_{elec} = -\frac{\partial E}{\partial w} = \frac{V_{in}^2}{2}\frac{\partial C_{eq}}{\partial d} = \frac{\varepsilon_0 A_C V_{in}^2}{2\left(d_e - w(\frac{L}{2},t)\right)^2} . \qquad (12)$$

Instead of using two calculations for $w(\frac{L}{2},t)$ and C_{eq} , mentioned above, it is possible to transfer the RF switch circuit ROM directly into MEMS system-level model, if use unique functional possibilities of the NetALLTED. The allow to introduce into an equivalent ROM circuit an additional element with the informative function (10), being arbitrarily connected to circuit nodes. Optimization procedures of NetALLTED allow to get the desirable values of this RF switch capacity and through its a desirable value of output signal of RF switch system-level model by the changing ROM parameters, which, in turn, are depended on the RF switch construction sizes and used material properties.

Separate models for the m-domain component are generated from FEM models and afterwards coupled using variables of the proper simulators. We need further development of order reduction techniques for very large nonlinear dynamic systems and automatic modeling of coupled field problem.

V. CONCLUSION

During MEMS design a system engineer will integrate the various sensing, computing and actuating elements into single chips by re-using existing designs for components. Complete MEMS can only be investigated at the higher abstraction levels such as schematic and system levels for which still accurate macromodels can be used. Creating such models by hand takes a lot of time and can lead to numerous mistakes and usually employs significant simplifications (only first and second degree of freedom). More valuable solution is automatic generation of such macromodels by extracting the necessary information from the detailed finite element models which have been built at the earlier design stages. This can be done by using model order reduction techniques which were considered in this paper. Then complete behavioural model of the whole MEMS can be compiled in VHDL-AMS language or presented as an equivalent circuit.