Macromodels of Micro-Electro - Mechanical Systems (MEMS)

Anatolii Petrenko A.I.

Abstract - Four automatic methods to generate MEMS reduced order macromodels , being based on full FEM/FDM models, are considered in this paper and compared.

1. FEM/FDM MODEL

On the way from ordinary Microelectronics to the Microsystems the silicon technology came in a new era, when the little moving structure elements of micron size are built into silicon and incorporated with electronics on a chip or in a multichips module. These structures have got the name of the **Micro-electro- mechanical systems (MEMS)**. MEMS typically involve multiple energy domains, such as kinetic energy, elastic deformation, electrostatic or magnetostatic stored energy and fluidic interactions. Much of the difficulty in the modeling of MEMS devices is due to the tight coupling between the multiple energy domains. Individual physical effects are governed by *partial differential equations* (PDE), typically nonlinear. When these equations become coupled, the computational challenges of highly meshed numerical simulation become formidable. The PDE can be numeral solved, applying the methods of finite differences (FDM) or finite elements (FEM).

By reading binary files of the FEM software like ANSYS it is possible to assemble a MEMS state-space model in the form of systems of first order ordinary diffential equations (ODE)

$$E_r z' + A_r z = B_r f\ , \qquad Y = C_r z \tag{1}$$

or systems of second order diffential equations

$$Mx'' + Dx' + Kx = Bf,\ Y = Q^T x + R^T x', \tag{2}$$

where A_r, E_r ,C_r,B_r, M ,D, K, B, C- are the system matrices, B_r,B are the input and the C_r,C -output matrices, f is input force. In mechanics matrices M , D and K are known as the ***mass, damping*** and ***stiffness*** matrices correspondingly.

In (1) the state space vector z is defined through the unknowns diflactions $u(x,t)$ and pressures $p(x,y,t)$ into the node points being automatically generated in MEMS structure :

$$z = [u_1 \ldots u_N \frac{\partial u_1}{\partial t} \ldots \frac{\partial u_N}{\partial t} p_{11} \ldots p_{MN}]^T \ . \tag{3}$$

By defining

$$E_r = \begin{bmatrix} D & M \\ M & 0 \end{bmatrix} \quad A_r = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \quad B_r = \begin{vmatrix} B \\ 0 \end{vmatrix} \quad C_r = \begin{vmatrix} Q \\ R \end{vmatrix} \quad z = \begin{vmatrix} x' \\ x'' \end{vmatrix} \tag{4}$$

second equations (2) can be transfer to the first (1).

The use of FEM/FDM codes in MEMS design is limited for two reasons [1]. First, the use of FEM/FDM codes to simulate MEMS devices is prohibitively cumbersome, expensive, and time consuming. Consequently, it is very expensive to close the loop on an FEM model of a device to allow for the design of feedback control laws or to use the model in system-level simulations. As a result, FEM/FDM models are mostly used to analyze the performance of finished devices rather than to design them. Second, FEM/FDM models use numerous variables to represent the device state. This approach makes the process of mapping the design space complex. Also, the relationship between each of these variables and the overall device performance is not clear to designers.

2. ROM (REDUCED ORDER MODEL)

It would be easier and more intuitive for the designer to explore the design space if the MEMS model had only a few variables with a clear relationship between them and the overall device performance. ***Reduced-order models(ROM)***, also called ***macromodels,*** lend themselves very well to these purposes. MEMS models resulted in seek to capture the most significant characteristics of a device behavior in a few variables governed by few ordinary-differential equations of motion (fig.1).

Anatolii Petrenko A.I.
Institute of the Applied System analysis of NTUU "KPI"

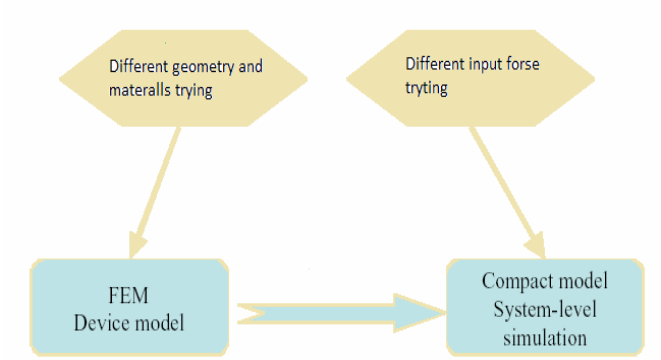

Fig.1. Use the reduced order model during MEMS design

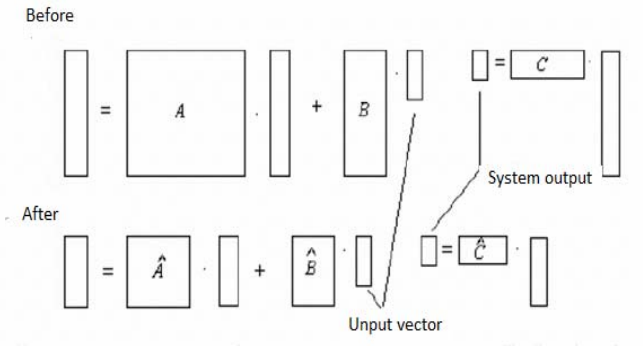

Fig.2. Reduced order model illustration

The variables can be selected to represent physically meaningful quantities. The resulting system is typically easy to simulate as a standalone model or integrate into system-level simulations. One of the approaches is to formally reduce the dimension of a system of ODEs (1) derived from a rigorous approach before integrating in time. The main idea behind the macromodel is that the number of ODE's needed to simulate the system has been reduced from perhaps many thousands in the case of the full FEM simulation, to just a few basis function coordinates. Thus the macromodel simulation can be very efficient computationally compared to the FEM model. One of the many popular methods for this kind of reduction is by using Krylov subspaces techniques.

2.1.Krylov/Arnoldi ROM

Since the order of the equation system (1) is quite large we need a lower order state-space model. Using Krylov/Arnoldi method, we reduce a state space equation

$$x'(t)=Ax(t)+Bf(t),\quad y(t)=C^T x(t) \tag{5}$$

to

$$x'_{red}(t)=A_{red}x_{red}(t)+B_{red}f(t),\quad y(t)=C_{red}{}^T x_{red}(t) \tag{6}$$

Sketch of MEMs model reduction before and after model reduction step is shown on fig.2. The dimention of matrices *A,B* and *C* and the internal state vectors are significantly smaller after model reduction for the same size of input vector *f* and output vector *y*. The node points being automatically generated in MEMS structure have to be partitioned into boundary nodes (related to the terminals at the model interface) and internal nodes (related to internal states of the reduced model (6).

Krylov's metod is based on approximation of system transfer function by moment matching.For linearized state-space model (5), perform a Laplace transform to obtain its frequency domain transfer function ***F(s)*** as

$$F(s)=C^T(sI-A)^{-1}B=-C^TA^{-1}(I-sA^{-1})^{-1}B \tag{7}$$

We can expand transfer function ***F(s)*** in Taylor series about some given s0 (in most applications s_0=0) and represent it as a polynomial in $(s\text{-}s_0)^i$, being expanded in Taylor series about s0=0:

$$F(s)=-C^TA^{-1}(I+sA^{-1}+s^2A^{-2}+\ldots\infty)B=\sum_{i=0}^{\infty}m_i s^i \tag{8}$$

where the *k*th coefficient of the Taylor series, $m_k=-C^TA^{-k}(A^{-1}B)$ is called the *k*th moment of the transfer function.

In terms of transfer function, the problem of constructing a reduced-order model of size *q* that approximates the input-output behavior of (5) can be stated as follows: determine the reduced linear state space system (6) such that the transfer function of the reduced system can approximate the transfer function of the original system [4].The transfer function of such a reduced system is

$$F_{red}(s)=-C_{red}{}^TA_{red}{}^{-1}(I+sA_{red}{}^{-1})^{-1}B_{red}=\sum_{i=0}^{\infty}m_{r,i}s^i \tag{9}$$

and its *k*th moment is

$$m_{r,k}=-C_{red}{}^TA_{red}{}^{-k}(A_{red}{}^{-1}B_{red}) \tag{10}$$

We can approximate $F(s)$ by $Fred(s)$ by matching the first q moments of $F(s)$, i.e. we will have $mr,k = mk$ for $k = 0, 1, \dots, q\text{-}1$. However, quite often practitioners use the Arnoldi process as it is more numerically robust and allows us to ***preserve stability and passivity*** of the original FEM model without extra computational cost [2]. To apply Arnoldi method to do moment matching for the system characterized by $\{A, B, C\}$ matrices, first define $A_1 = A^{-1}$ and $b = A^{-1}B$, and then apply Arnoldi method on A_1, b and q. The results are Hq and Lq. With these operations, the kth moment of the system (5) can rewritten as

$$m_k = -C^T A^k (A^{-1}B) = C^T A_1^k b = \|b\|_2 C^T L_q H_q^k e_1 = \|b\|_2 C^T L_q H_q^{k+1} H_q^{-1} e_1 \quad (11)$$

From comparison of (10) and (11) it can be seen that if we choose $A_{red} = Hq^{-1}$, $B_{red} = \|b\|_2 H_q^{-1} e_1$ and $C_{red} = L_q^T C$, then $m_k = m_{r,k}$ (k=0,1,…,q-1) can be satisfied.

In the Arnoldi method, $L_q = [l_1 \dots l_q]$ is an orthonormal basis of the Krylov subspace ($l_1 = b / \|b\|2$) and $Hq = (h_{i,j})$ is a matrix representation of the original system A on the Krylov subspace with respect to L_q. These two matrices have the following properties: $L_q^T L_q = 1$ and $AL_q = L_q H_q + h_{q+1,q} I_{q+1} e_q^T$,where e_q is the qth standard unit vector and $h_{q+1,q}$ is a scalar [2].

For example,the mentioned model reduction has been successfully used during geometry optimization of the microaccelerometer, microgyroscope and radio frequency micro-switch [3]. Using Krylov/Arnoldi approach, only a postprocessor is necessary to generate a system-level model in one of the well established model description languages: pure C code, HDL-A , MAST and the new standardized VHDL-AMS which are supported by powerful system simulators. They will be used more and more in other physical disciplines because they offer description means for mechanical, thermal, and other non-electrical systems.

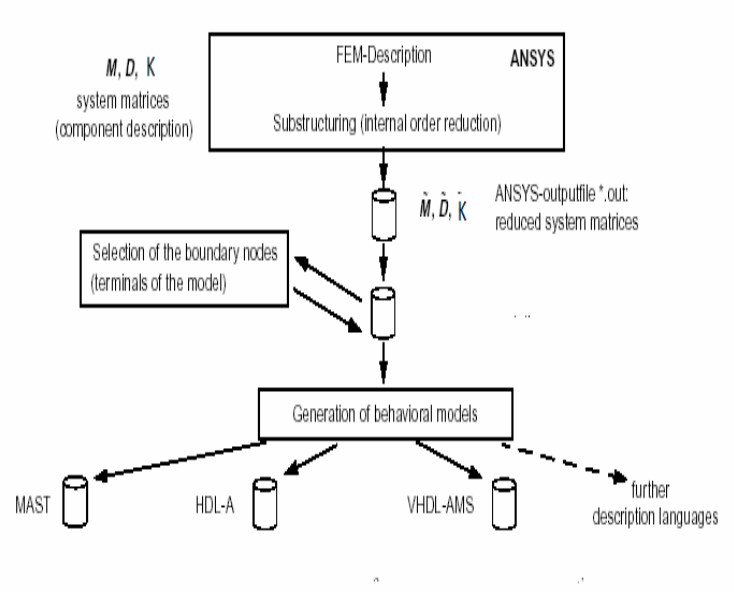

Fig.3. An ANSYS postprocessor for reduced order model generating

2.2.Modal decomposition ROM

Some FEM simulators (e.g. ANSYS) have build-in algorithms for order reduction and offer the opportunity to export the reduced model equations. Using a weighted sum of n mode shapes (modal amplitudes q_i, eigenvectors $\varphi_i(x, y, z)$) of the mechanical structure it is possible to represent its deflection u as:

$$u(x,y,z) = u_{eq} + \sum_{i=1}^{n} q_i(t)\varphi_i(x,y,z), \quad (12)$$

where u_{eq} is the initial diflaction caused by prestress in equilibrium state. Especially for MEMS a few eigenmodes are usually enough to accurately describe the dynamic response of the structure [4]. The strain energy W_{mech} that is stored within the structure due to deflection or prestress is expressed as a function of the modal amplitudes qi. Geometrical nonlinearities and stress-stiffening are considered by calculating the modal stiffness k_{ij} from thesecond derivatives of the strain energy with respect to the modal amplitudes:

$$k_{ij} = \frac{\partial^2 W_{mech}}{\partial q_i \partial q_j} \quad (13)$$

The modal masses m_i and modal damping constants d_i are calculated from the eigenfrequencies ω_i of the modes i and the entries of the modal stiffness matrix k_{ij} :

$$m_i = \frac{k_{1i}}{\omega_i^2} \qquad d_i = 2\xi_i \omega_i m_i \quad (14)$$

where ξ_i is the modal damping ratio of mode i. The modal damping ratios represent the fluidic damping of the structure and can be obtained from analytical calculations (squeeze or slide film damping), numerical fluid dynamic simulations or measurements. The deflection of the mechanical structure changes the capacitances between the electrodes in a nonlinear manner. The capacitance C_{op} between the electrodes *o* and *p* is calculated as a function of the modal amplitudes and therefore provides the coupling between the mechanical and electrical quantities.

The displacement current I_o through the electrode o can be calculated from the stored charge

$$I_0 = \frac{dQ_0}{dt} = \sum_p \frac{d(C_{op}(q1...qn)(V_0 - V_p))}{dt} \qquad (15)$$

where Vo and Vp are the voltages at the electrodes. The governing equation describing the whole electrostatically actuated micromechanical structure in terms of modal coordinates is:

$$F_{M,i} = m_i q_i'' + d_i q_i' + \frac{\partial W_{mech}(q1..qn)}{\partial q_i} + \sum_j \varphi_{ij}\lambda_j + \sum_r \frac{\partial C_{op}^{(r)}(q1...qn)}{\partial q_i}(V_0 - V_p)^2 \qquad (16)$$

where $F_{M,i}$ is the modal force and r the number of capacitances involved between the multipleelectrodes. The λ_j are the reaction forces to the external forces $F_{N,j} = -\lambda_j$ at the master nodes *j* of the FEM mesh that remain accessible in the behavioral model. The reduced-order model (ROM) consists of the equations (12), (15) and (16), which fully describe the static and dynamic nonlinear behavior of the flexible structure and its nonlinear coupling to the electrostatic domain. All missing ***parameters of the ROM can be derived from a detailed fully coupled FEM model*** of the MEMS component in a highly automated manner. The eigenvectors φ_i and eigenfrequencies ω_i of the considered modes *i* are taken from the modal analysis of the mechanical structure. The shape function of the strain energy $W_{mech}(q_i)$ as well as the functions of the capacitances $C_{op}(q_i)$ are expressed in a polynomial form. They are fitted to a set of sample points of strain energy and capacitances extracted from a series of static analyses of the FEM model in which the structure is deflected to various linear combinations of its mode shapes (in collocation points or as least-squares procedure). This ROM-Tool available in ANSYS/Multiphysics since Release 7 is one implementation of this method.

2.3.Galerkin's approximated ROM

As in the previous case the desired PDE solution u(x,t) can be approximated by series expansion of time varying coefficients $\alpha_i(t)$ and spatially varying basis functions $\alpha_i(x)$:

$$\hat{z}(x,t) = \sum_{i=1}^{N} \alpha_i(t)\alpha_i(x) \qquad (17)$$

where $\hat{z}(x,t)$ is the approximation for $z(x,t)$ and consists of the time histories of both pressure $\hat{p}(x,y,t)$ and diflaction $\hat{u}(x,t)$. As it is well known for the Galerkin's method the PDE the residual ($L(\hat{u}) - f$) had be orthogonal to each $\alpha(x)$ of the basis functions in *H*:

$$(\alpha_i, L(\hat{u}) - f) = \int \alpha_i^T (L(\hat{u}) - f)dx = 0, \quad i = 1, N, \qquad (18)$$

where *L* is a differential operator (possibly nonlinear), and *f* is a input vector.

The choice of orthogonal basis functions $\alpha(x)$ can be done as the following. First the MEMS dynamics are simulated using a slow but accurate technique such as FEM or FDM. A sets of runs may be used to suitably characterize the operating range of the device. The spatial distributions of each state variable u(x,t) are then sampled at a series of N_s different times during these simulations, and the sampled distributions are stored as a series of vectors, $\{u_i\}$, where each corresponds to a particular "snapshot" in time. Now suppose we would like to pick orthogonal basis N functions $\{\alpha_1,\alpha_2,\ldots,\alpha_N\}$ in order to represent the observed state distributions as closely as possible. One way to do this is to attempt to minimize the following quantity:

$$\sum_{i=1}^{N_a} [u_i - proj(u_i, span\{\alpha_1,...,\alpha_N\})]^2 \qquad (19)$$

where $proj(v,W)$ is the projection of the vector v onto subspace W . In other words, we minimize a least squares measure of the "error" distances between the observed states and the basis function representation of those states. It turns out that this can be accomplished quite simply by taking the SVD (Singular Value Decomposition) of the matrix U, whose columns are .u_i. The SVD of is given as follows:

$$U = V\Sigma W^T \qquad (20)$$

where $\Sigma = diag(\sigma_1,...,\sigma_N)$ is a diagonal matrix, V and W are orthonormal matrices of eigenvectors, , and $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_N \geq 0 \quad N < N_a$ It was shown that the proper orthogonal basis functions minimizing (19) can be chosen by setting $a_i = v_i \quad for \quad i = \{1,2,...,N\}$ for where v_i is the columns of V. SVD decompositions have previously been used for generating basis functions for image compression and analysis [5]. However it still takes long time to directly recompute the time-dependant nonlinear terms in macromodels, which are generated by proper orthogonal

decomposition (POD) method with Galerkin's procedure, at every time step during the macromodel simulation.

2.4.Circuit ROM

Taking in to account relations between displacements *x*, velocities *v* and accelerations *a* : $a = dv/dt, \quad x = \int v dt$, it is possible to present an equation (2) in the form

$$\frac{d}{dt}(Mv) + Dv + \int Kv dt = F(t) \qquad \text{or}$$

$$\tilde{C}\dot{v} + \tilde{G}v + \tilde{L}v = F(t)\ , \qquad (21)$$

where $\tilde{C} = M, \tilde{G} = D, \tilde{L} = K$ – are equivalent matrices of capacitances, conductance and inductances. The elements of matrices *C, G, L* are formed from the elements of the mass, damping and stiffness matrices in the following ways [6]:

$$C_{ij} = -m_{ij},\ i,j = \overline{1,N},\ i \neq j;\ C_{ii} = \sum_{j=1}^{N} m_{ij},\ i = \overline{1,N};\ L_{ij} = -1/k_{ij},\ i,j = \overline{1,N},\ i \neq j;$$

$$L_{ii} = 1/\sum_{j=1}^{N} k_{ij},\ i = \overline{1,N};\ G_{ij} = -d_{ij},\ i,j = \overline{1,N},\ i \neq j;\ G_{ii} = \sum_{j=1}^{N} d_{ij}, i = \overline{1,N}.$$

where N is a number of equations or nodes of the MEMS structure. In this way a capacitance-inductance-conductance circuit model shall be constructed which reflects correctly mass, damping and stiffness matrices (fig.4). For reduction of such model size the Y/Δ transformation is usually used by removing "high frequencies" inner nodes with the maximum value of time constant. In this case (as in the previous ones) it is necessary to find a compromise between the model reduction order and its exactness (adequacy).

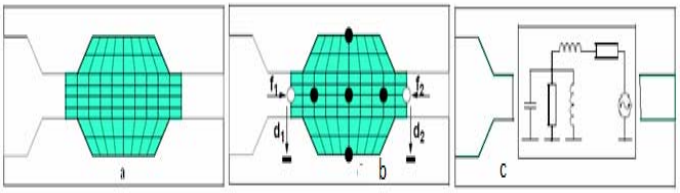

Fig.4. Illustration of transition procedure from the model of canonical equations of finite element (a) through the model of the reduced order (b) to the behavior macromodel (c)

The application of Y/Δ transformation does not require modification of existent silulation tool , as end results is a reduced circuit unlike the existing approaches (for example, in CoventorWare), where it is represented by the reduced system of differential equations. Thus, the design of electric and nonnelectric MEMS components can be done by one circuit simulation, for axample, by NetALLTED domestic circuit simulation package and making use of its unique power optimization and tolerancing procedures [7,8].

3. MEMS system-level model

Model size reduction can produce a compact model for system-level simulation and the latter can be used without changes for different input functions. In this case, one obtain enormous saving in the computational time. However, quite often an engineer would like to change the geometry or materials properties of the device in order to optimize its performance. In this case, the reduced order model (ROM) should be generated again after the change is incorporated in the original finite element model. In this case, the advantage of model reduction is clearly depends on the computational cost to generate the reduced model.

This approach is illustrated by the example of design of a capacitive RF Switch (fig.5) with such cyclic organization of calculations .

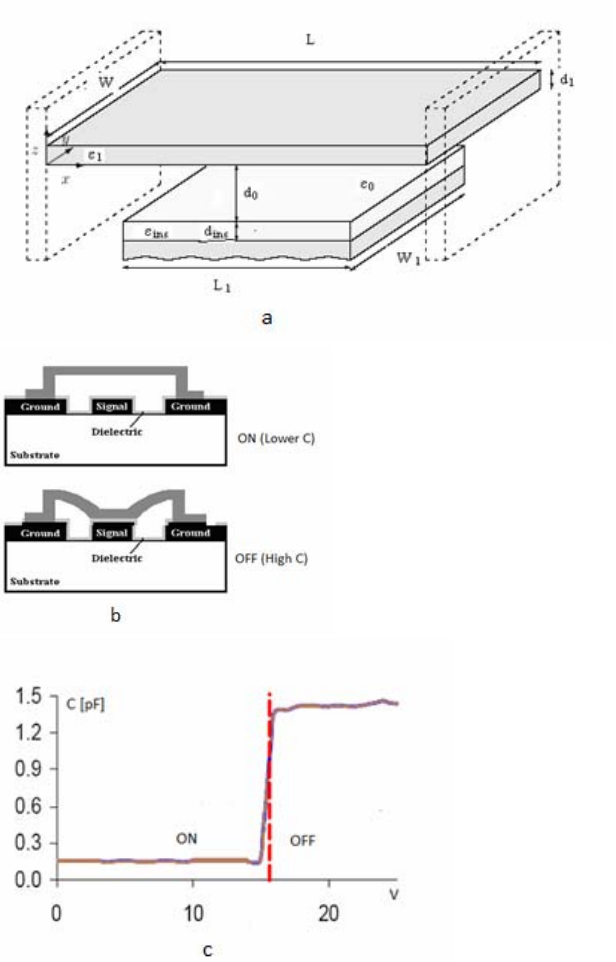

Fig.5. Simplified structure of a doubly supported capacitive RF Switch (a), its capacitive model (b) and way of performance (c)

At system (circuit) level MEMS is modelling with the capacity macromodel of RF switch and a voltage value V, applied to a capacitor, being determined for the particular

time point or for certain frequency. So the switch in hand may be simulated by nonlinear capacitance:

$$C_{eq} = C_o + (C_{L/2} - C_0)(1 - e^{-\tau t}), \qquad (22)$$

where C_0 is the smallest capacitance in the absent of voltage V:

$$C_0 = \frac{\varepsilon_0 A_C}{d_e}, \qquad (23)$$

$C_{L/2}$ is the largest capacitance when a plate center displacement is calculated from the ROM macromodel equations:

$$C_{L/2} = \frac{\varepsilon_0 A_C}{d_e - w(\frac{L}{2}, t)}, \qquad (24)$$

where A_C is the bottom electrode area; d_e is an equivalent gap ($d_e = d_0 + \frac{d_1}{\varepsilon_1} + \frac{d_{ins}}{\varepsilon_{ins}}$); ε_0 is the absolute dielectric permittivity of the vacuum, ε_1 is the relative dielectric permittivity of the poly silicon , ε_{ins} is the relative dielectric permittivity of the insulator; w is a plate diflaction . The most large value of this capacitance corresponds to the value $w_{\max} = d_0$. The electrostatic force acting on the capacitor surfaces is the Coulomb force:

$$\tilde{F}_{elec} = -\frac{\partial E}{\partial w} = \frac{V_{in}^2}{2}\frac{\partial C_{eq}}{\partial d} = \frac{\varepsilon_0 A_C V_{in}^2}{2\left(d_e - w(\frac{L}{2}, t)\right)^2} \qquad (25)$$

As the displacement $w(\frac{L}{2}, t)$ reaches the value $w_{\max} = d_0$, the hard stop will restrict its further increase, but input voltage V_{in} and $\tilde{F}_{elec}$ can be further increased. Assume that the input voltage V_{in} is a superposition of a constant voltage V_{DC} and a time dependent signal $V(t)$ and that $V_{DC} >> V(t)$. The elastic-plastic properties of the points of contact are simulated by a spring with rigidity c and a damper of damping factor b_0. When the beam center moves past $w_{\max}$, it starts interacting with the spring that represents the contact. The damper is introduced to take into account the energy dissipation at the contact. The following equations are used to represent this model:

$$R = \begin{cases} 0 & with \quad w \le w_{\max}; \\ c\left(w_{\max} - w\right) - b_0 w' & with \quad w > w_{\max}, \end{cases} \qquad (26)$$

where R is the interaction force.

The process of interaction simulated by the force R sometimes can be highly sensitive to the values of the model parameters (rigidity and damping factor), especially if $w_{\max} \approx d_e$. The interaction can induce high-frequency motions and slow down the rate of convergence considerably. Therefore, a good deal of attention must be paid to accurately model and represents this process using experimental data.

If the input voltage is increased more, there will be no equilibrium and the plate collapse takes place (fig.5,b). In this case, a hard stop or some other arrangement must be introduced to limit the plate motion. During the plate collapse, the difference between the electrostatic force and the elastic force of the spring will continue to increase. Therefore, when the plare drops down into the hard stop, it is not enough to reduce the input voltage below to release the plate. The input voltage should be reduced more to make the electrostatic force at least equal to the elastic force. Hence, the plate capacitance exhibits the hysteretic behavior .

But if the insulation layer is rather thick its restrictive effect should be taken into account. If the maximal center displacement $w_{\max}$ is equal to initial thickness air gap:

$$w_{\max} = d_0. \qquad (27)$$

the plare center touches the insulator top surface when the input DC voltage reaches the value of $V_{\max}$ ($V_{\max}$ may be calculated from simulation). But as soon as the input voltage will be decreased under this value the plate will leave the hard stop. So, it does not demonstrate the hysteretic behavior (fig.5,c).

Parameter τ in (22) can be calculated through the beam center displacement $w(\frac{L}{2}, t)$ and its velocity $v(\frac{L}{2}, t) = w'(\frac{L}{2}, t)$, which are defied from equations (21) in the following way:

$$\tau = \frac{1}{3} w(\frac{L}{2}, t) / v(\frac{L}{2}, t). \qquad (28)$$

The coefficient 3 appears in (28) due to the fact that a capacitance recharge to 98% for a time value which is equal approximately 3τ.

By summarizing above mentioned the following procedure is proposed:

▪ For particular MEMS its geometrical structure is described and full FEM/FDM simulation is done (say, by ANSYS) , resulting in receiving the initial equations (1).

▪ The equivalent model equations (21) are built then and a designer had to partition a complex equivalent model circuit into smaller ones and had to define special nodes as a basis for order reduction. Later, these nodes have to be partitioned into *boundary nodes* (related to the terminals at the model interface) and internal nodes (say, related to the centre of a beam or a membrane).

▪ The procedure of Y/Δ –transformation is applied to exclude internal nodes and reduced equations are solved to get necessary values of the centre plate (membrane) diflaction $w(\frac{L}{2},t)$ and its velocity $v(\frac{L}{2},t)$.

▪ Using these quantities and equations (22)-(26), the user generates a MEMS system-level model .

It is possible to see two included procedures: one for development of ROM for the RF switch plate, where a diflaction and speed of diflaction of central point of swith plate is calculated for the value V, and second - for determing parameters of MEMS system-level model and updating an equivalent capacity value , using values of co-ordinates of plate central point. Then the cycle of calculations recurs whereupon.

Instead of using two procedures mentioned above it is possible to transfer RF switch circuit ROM directly into MEMS system-level model, if introduce in it the additional arbitrarily connected element with an informative function which is determined by equation (26) as functional possibilities of the circuit simulator NetALLTED (ALL TEchnologies Desinger) [9]. Optimization procedures of NetALLTED allow to get the desirable values of this RF switch capacity and through its a desirable value of outputl signal of RF switch system-level model by the changing ROM parameters , which, in turn, are depended on the RF switch constraction sizes and used material propertities.

Conclution

Four automatic methods to generate MEMS reduced order macromodels , being based on full FEM/FDM models, were demonstrated in this paper. Three of them are suitable for simulators with possibilies to get input information in the equation forms (ODEs or OAEs) [10]. The forth one in opposite produces macromodels in circuit presentation and so it is more suitable for circuit simulators. It seems to be interesting and perspective to try combination of mentioned approaches, say, at the beginning to use Krylov/Arnoldi reduction of ODE demention ,then to build the proper equivalent circuit presentation for such decreased ODE systems and finally to apply Y/Δ transformation for further reducing macromodel order.

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